Reduced Mass-Weighted Proper Decomposition of an Experimental Non-Uniform Beam

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Abstract

The goal of this research is to estimate the mode shapes of a non-uniform beam using sensed displacements, via accelerometers, at regular intervals along the beam. This body of work continues the research of Yadalam and Feeny [1] on reduced mass-weighted proper orthogonal modal decomposition for modal analysis. In regular proper orthogonal decomposition (POD), the ensemble matrix **X**, typically built from measured displacements, is used to construct the correlation matrix $\mathbf{R}=1/N\ \mathbf{X}\mathbf{X}^T$, where N is the number of samples taken of the displacement vector **x**, where $\mathbf{x} = [x_1\ x_2\ \ldots\ x_M]^T$, and M is the number of sensors. Then each row of **X** is the time history of each sensor. For example, the first row is $\mathbf{x_1}= [x_1(0),\ x_1(\Delta t),\ x_1(2\Delta t),\ \ldots,\ x_1(N\Delta t)]^T$. The proper orthogonal modes (POMs) are computed by solving the eigensystem $\mathbf{Rv}=\lambda\mathbf{v}$ [2, 3]. For a uniform, lightly damped, free vibration system, the POMs resemble the linear normal modes (LNMs) [4]. For non-uniform systems the weighted eigenvalue problem $\mathbf{RMv}=\lambda\mathbf{v}$, where **M** is the mass matrix, produces estimates of the linear normal modes. Since the dimension of the mass matrix of the beam is large compared to the dimensions of **R**, interpolating functions can be used to compute a reduced-mass matrix $\mathbf{M}_r$. Then the eigensystem $\mathbf{RM_r v}=\lambda\mathbf{v}$ is solved. The eigenvectors correlate to the linear normal modes, and the eigenvalues relate to spectrum energy density of those modes.

In this work, the free-response displacement time histories of a cantilevered saw blade (a non-uniform, thin steel, cantilevered beam) were obtained by integrating M = 11 sensed accelerometer signals, sampled at the time interval Δt = 0.0002 sec, and then used to build the ensemble matrix **X**. The beam was 11 inches long, 0.01 inches thick, and the width varied from 1 inch at the tip to four inches at a distance of 2 inches from the clamp. The beam was clamped so that its centerline was horizontal, and flexural displacements were horizontal. The matrix $\mathbf{M}_r$ was computed by integrating linear interpolating tent functions through the mass distribution of the beam. A fast Fourier transform of an accelerometer signal indicated modal frequencies of 8.545 Hz, 40.28 Hz, 107.4 Hz, 205.1 Hz, 498 Hz, and 677.5 Hz. The accelerometer behavior was phase-distorted near the first (lowest) modal frequency of the beam. Therefore, a high-pass filter of 20 Hz was applied, removing this distortion, but consequentially removing the activity of the first mode. The mass-weighted POD, as well as the regular POD for comparison, was performed on the filtered data. Figure 1 shows the extracted shapes of the second, third, fourth and fifth flexural modes of the beam. In the figure, the blue (Δ) curves are the mode shapes from the mass-weighted POD, the green (□) curves are from the regular POD, and the red (O) curves are from a finite element analysis, and are regarded as approximations to the LNMs. The

second mode is dominant in the signals, and expected to be the closest fit for both mass-weighted and regular POD. The third mode shows high accuracy with the mass-weighted POD. The next modes are satisfactory, but tend to have accumulated error from the previous modes.

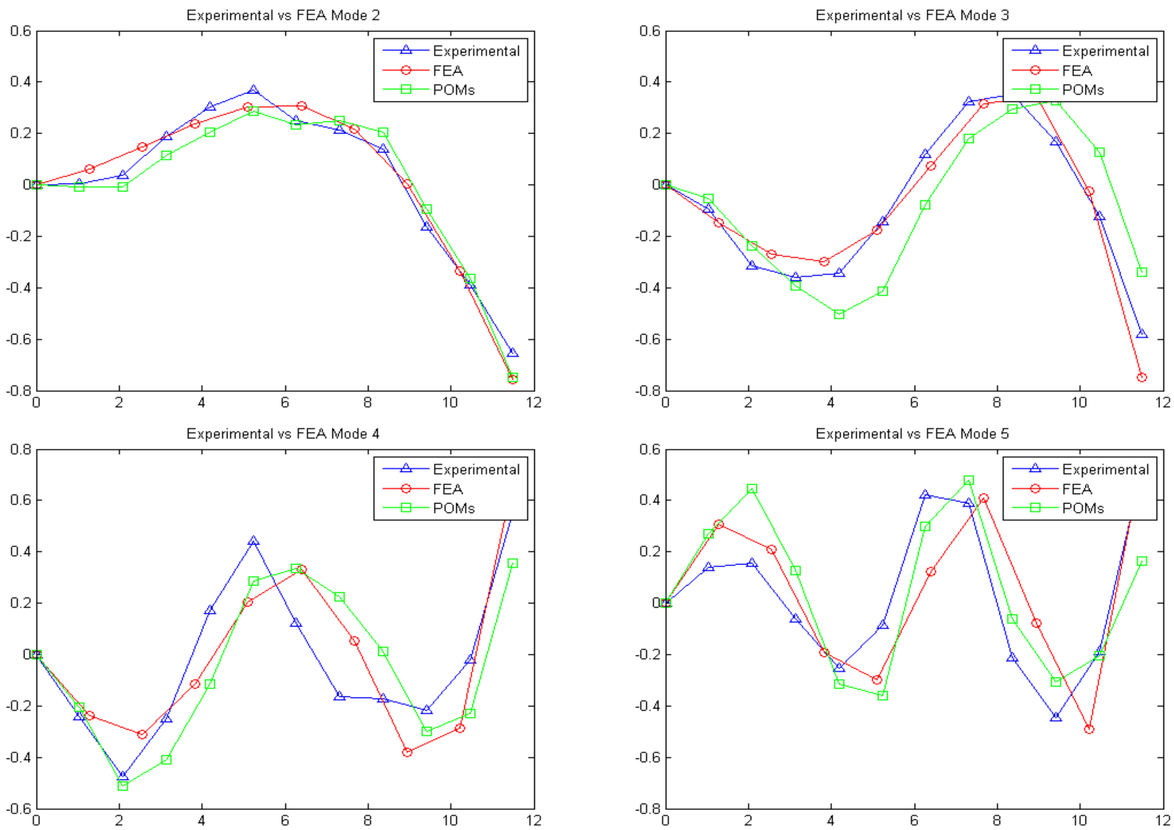

Figure 1. Plots of mode estimates for the 2nd , 3rd, 4th and 5th modes. The blue (Δ) curves are the mode shapes from the mass-weighted POD, the green (□) curves are from the regular POD, and the red (O) curves are from a finite element analysis.

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